Sanjay Ghodawat University, Kolhapur

2020-21

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

EXM/P/09/00

Year and Program: S.Y. B. Sc. **School of Science** **Department: Mathematics**

Course Code: MTS204.2 **Course Title: Number Theory** **Semester : IV**

Day and Date: **End Semester Examination** **Time : 2 Hrs.**
Max Marks : 50

Instructions: 1) All Questions are compulsory.
2) Use of non-programmable calculator is allowed.
3) Assume standard notations whenever necessary.

		Marks	Bloom's Level	CO
Q.1	**Solve the following (any two)**			
a)	Show that, $1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$, for positive integer n.	8	L3	CO1
b)	Find $\gcd(12378, 3054)$. Also express $\gcd(12378, 3054)$ in linear combination of 12378 and 3054.	8	L3	CO1
b)	For any positive integers a and b, prove that $\gcd(a,b) \times \text{lcm}(a,b) = ab$.	8	L3	CO1
Q.2	**Solve the following (any two)**			
a)	If p is a prime and $p \mid a_1 a_2 \cdots a_n$, then prove that $p \mid a_k$ for some k, where $1 \le k \le n$.	8	L1	CO2
b)	Find the remainders when 2^{50} and 41^{65} are divisible by 7, by using linear congruence.	8	L1	CO2
c)	For arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same non negative remainder when divided by n.	8	L1	CO2
Q.3	**Solve the following (any two)**			
a)	Solve the linear congruence $9x \equiv 21 \pmod{30}$.	9	L2	CO3
b)	With usual notation show that the functions τ and σ both are multiplicative functions.	9	L2	CO3
c)	If n is positive integer and $\gcd(a,n) = 1$, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.	9	L2	CO3

*****